

## C3.Q103.NOTES: 14B Limits and Optimization

### LESSON 1 (14.7)

#### Definition:

Let  $f$  be a function of two variables. A pair  $(a,b)$  is a critical point of  $f$  if either

- (i)  $f_x(a,b) = 0$  and  $f_y(a,b) = 0$  or
- (ii)  $f_x(a,b)$  or  $f_y(a,b)$  does not exist

#### Definition:

Let  $f$  be a function of two variables that has continuous second partial derivatives. The

discriminant  $D$  of  $f$  is given by  $D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - [f_{xy}(x,y)]^2$

#### Test for Local Extrema:

Let  $f$  be a function of two variables that has continuous second partial derivatives throughout an open disk  $R$  containing  $(a,b)$ . If  $f_x(a,b) = f_y(a,b) = 0$  and  $D(a,b) > 0$ , then  $f(a,b)$  is

- (i) a local maximum of  $f$  if  $f_{xx}(a,b) < 0$
- (ii) a local minimum of  $f$  if  $f_{xx}(a,b) > 0$

#### Theorem:

Let  $f$  have continuous second partial derivatives throughout an open disk  $R$  containing  $(a,b)$ . If  $f_x(a,b) = f_y(a,b) = 0$  and  $D(a,b)$  is negative, then  $P(a,b, f(a,b))$  is a saddle point on the graph of  $f$ .

Example 1: Let  $f(x,y) = 1 + x^2 + y^2$ , with  $x^2 + y^2 \leq 4$ . Find the extrema of  $f$ .

Example 2: If  $f(x,y) = y^2 - x^2$  and the domain of  $f$  is  $\mathbb{R}^2$ , find the extrema of  $f$ .

Example 3: If  $f(x,y) = x^2 - 4xy + y^3 + 4y$  find the local extrema and saddle points of  $f$ .

Example 4: If  $f(x,y) = x^2 - 4xy + y^3 + 4y$ , find the extrema of  $f$  on the triangular region  $R$  that has vertices  $(-1,-1)$ ,  $(7,-1)$  and  $(7,7)$

Example 5: If  $f(x,y) = \frac{x^3}{3} + \frac{4}{3}y^3 - x^2 - 3x - 4y - 3$  find the local extrema and saddle points of  $f$ .

Example 6: A rectangular box with no top is to be constructed to have a volume of  $12\text{ft}^3$ . The cost per square foot of the material to be used is \$4 for the bottom, \$3 for two of the opposite sides, and \$2 for the remaining pair of opposite sides. Find the dimensions of the box that will minimize cost.

**LESSON2 and LESSON3**

*You will need extra notebook paper for lessons 2 and 3 section 14.7 and 14.2.*